Electroweak spin gauge theories and the frame field

1987 J. Phys. A: Math. Gen. 20 6561

(http://iopscience.iop.org/0305-4470/20/18/052)

J. Phys. A: Math. Gen. 20 (1987) 6561-6580. Printed in the UK

Electroweak spin gauge theories and the frame field

J S R Chisholm† and R S Farwell‡

† University of Kent, Canterbury, Kent CT2 7NZ, UK
‡ St Mary's College, Twickenham, Middlesex, UK

Received 18 May 1987, in final form 16 July 1987

Abstract. The principles of spin gauge theories are explained. A particular spin gauge symmetry within the Clifford algebra $C_{2,6}$ is shown to give the correct GSW electroweak interactions for the electron-neutrino system. A new concept of mass is introduced, the electron mass being interpreted as an interaction with the 'frame field', which is proportional to the spacetime dependent Dirac matrices $\{\gamma^\mu(x)\}$. Including the frame field in the 'extended covariant derivative' Δ^μ and calculating $[\Delta^\mu, \Delta^\nu]$ gives, along with the boson Lagrangian kinetic terms, exactly the correct photon, W and Z mass matrix. Transformation of the lepton extended covariant derivative to the 'quark representation' of the Clifford algebra, which is determined by the electromagnetic coupling constants, reproduces the GSW interactions for the up and down quarks. Thus, for the first generation electroweak theory, the 'Higgs-Kibble mechanism' is replaced by the frame field concept of mass. The models studied indicate that an energy associated with the frame field is approximately three times the W boson rest mass M_W. A refinement of the theory suggests a fermion mass of the order of M_W.

1. The spin gauge theory principle

A spin gauge theory is a Lagrangian field theory. As in standard gauge theories, the form of the Lagrangian density is determined by requiring invariance under certain local (that is, spacetime dependent) transformations. However, spin gauge theories [1-3] differ from standard theories in recognising that spinors are elements of Clifford algebras, and in requiring that all elements of an algebra be subjected simultaneously to a gauge transformation. In this section, we exemplify the spin gauge principles by treating the free Lagrangian density L_0 for a Dirac bispinor, describing an electron.

We represent the electron by a four-component column vector $\varepsilon(x)$, where $x = \{x^\mu, \mu = 1, 2, 3, 4\}$ is the set of spacetime coordinates, and define a set of 4×4 matrices $\{\gamma_\mu, \mu = 1, 2, 3, 4\}$ to be a representation of the Dirac matrices. In later sections we also use a four-component column vector $\nu(x)$ representing the neutrino bispinor. The 'bar' or Hermitian conjugate of any four-component bispinor ψ is given by $\bar\psi = \psi^+\gamma$, where the 4×4 conjugation matrix γ is defined by $\gamma_\mu^+ = \gamma\gamma_\mu\gamma^{-1}$ and is such that $\gamma^+ = \gamma$.

The set of Dirac matrices $\{\gamma_\mu, \mu = 1, 2, 3, 4\}$ is a representation of the set of generators of the Dirac algebra, which is the real four-dimensional Clifford algebra $C_{1,3}$ associated with spacetime. The set $\{\gamma_\mu\}$ represents the set of vectors of the algebra. The inner product of these vectors defines the Minkowski spacetime metric through the basic anticommutation relations

$$\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2g_{\mu\nu}I \tag{1.1}$$

where $(g_{\mu\nu}) = \mathrm{diag}(-1, -1, -1, 1)$ and I is the 4×4 identity matrix. The full algebra is $\{\gamma_A; A=1,2,\ldots,16\} \equiv \{I, \gamma_\mu, \gamma_{\mu\nu}, \gamma_{5\mu}, \gamma_5; \mu, \nu = 1,2,3,4, \mu<\nu\}$, where $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$, $\gamma_{5\mu} = \mathrm{i}\gamma_5\gamma_\mu$ and $\gamma_{\mu\nu} = \mathrm{i}\gamma_\mu\gamma_\nu$ [4].

Dirac bispinors are elements of the minimal left ideal of the algebra $C_{1,3}$. The geometrical significance of representing bispinors as elements of the minimal left ideals of the Dirac algebra was discussed by Kähler [5]. Elements of the minimal left ideal are 4×4 matrices having non-zero entries in one column only. The column vector $\varepsilon(x)$ is then identified with this non-zero column of the matrix. Since the Dirac spinors and the Dirac matrices are both expressed in terms of a representation of the algebra $C_{1,3}$, if we choose to change the representation of the spinor by performing a gauge transformation, then we should also change the representation of the matrices in a corresponding way. Since $\varepsilon\bar{\varepsilon}$ is a 4×4 matrix, and $\varepsilon\bar{\varepsilon}$ and the matrices γ_μ are all elements of the same algebra, the requirements of algebraic consistency suggest that the matrices γ_μ should transform in the same way as $\varepsilon\bar{\varepsilon}$. The requirement that $\{\gamma_\mu\}$, as well as ε and $\bar{\varepsilon}$, transform under gauge transformations is the fundamental difference between standard and spin gauge theories.

The free electron Lagrangian density is

$$L_0 = \tfrac{1}{2}[\bar{\varepsilon}\mathrm{i}\gamma_\mu(\partial^\mu\varepsilon) - (\partial^\mu\varepsilon)^+\gamma\mathrm{i}\gamma_\mu\varepsilon] - m\bar{\varepsilon}\varepsilon \tag{1.2}$$

where m is the mass of the electron. This is invariant under the global spin gauge transformations

$$\gamma_\mu \to R\gamma_\mu R^{-1} \tag{1.3a}$$

$$\varepsilon \to R\varepsilon \qquad \bar{\varepsilon} \to \bar{\varepsilon}R^{-1} \tag{1.3b}$$

$$R = \mathrm{e}^{-\mathrm{i}\theta} \tag{1.3c}$$

where θ is any element of $C_{1,3}$ and in general is a Clifford number

$$\theta = \theta_0 I + \theta^\mu\gamma_\mu + \theta^{\mu\nu}\gamma_{\mu\nu} + \theta^{5\mu}\gamma_{5\mu} + \theta^5\gamma_5.$$

However, if R is x-dependent, that is, if we consider a local gauge transformation (1.3), then

$$L_0 \to L_0 + \tfrac{1}{2}[\bar{\varepsilon}\mathrm{i}\gamma_\mu R^{-1}(\partial^\mu R)\varepsilon - \varepsilon^+(\partial^\mu R^+)(R^+)^{-1}\gamma\gamma_\mu\varepsilon]$$

and the invariance of the Lagrangian is lost. Invariance can be restored by replacing the derivative ∂^μ in (1) by a covariant derivative

$$D^\mu = \partial^\mu - \Omega^\mu(x)$$

where Ω^μ belongs to the algebra $C_{1,3}$. The resultant Lagrangian L_1 is given by

$$L_1 = L_0 + \bar{\psi}\mathrm{i}\gamma_\mu\Omega^\mu\psi \tag{1.4}$$

and is invariant under x-dependent transformations (2) provided that

$$\Omega^\mu \to R\Omega^\mu R^{-1} + R(\partial^\mu R^{-1}). \tag{1.5}$$

A study of this full group of transformations has been carried out by McEwan [6]. We note that the original Weyl gauge theory of electromagnetism corresponds to taking $\theta(x) = \theta_0(x)I$ simply; then R commutes with γ_μ, which are therefore unchanged by the transformation (1.3a). So the Weyl theory is both a standard and a spin gauge theory.

This exemplifies the basic principle of a spin gauge theory: that we gauge the freedom to choose a different representation of both spinors and spin (γ) matrices at each point x in spacetime. The set of matrices $\{\gamma_\mu\}$ appearing in the Lagrangian (1.4) are in general x-dependent and hence constitute the components of a matrix field. Since γ_μ are the basis vectors in the Clifford algebra of spacetime, we refer to γ_μ as the components of the frame field.

Our original aim in proposing such a gauge theory was that it should provide a framework for the unification of the strong, weak and electromagnetic interactions in which the three interactions would be associated with different aspects of the same symmetry principle [4, 7, 8]. In this paper we present a simplified model describing the Glashow-Salam-Weinberg (GSW) electroweak interactions of the first generation of fermions [9]. A single bispinor cannot be used, since the weak interactions mix the spinors of the electron and its neutrino. A multispinor consisting of several spinors is formed and the Clifford algebra enlarged accordingly. Generally, if the multispinor has 2^n components then a Clifford algebra $C_{p,q}$ $(p+q=2n)$ is used. This Clifford algebra is associated with a $2n$-dimensional inner product space. We choose to relate the first four basis vectors of this $2n$-dimensional space to $\{\gamma_\mu\}$, so that four-dimensional spacetime can be thought of as embedded in the higher dimensional space. However, the multispinors are assumed to be functions of the spacetime coordinates x only, and we do not introduce coordinates corresponding to the remaining $(2n-4)$ dimensions. It might be possible to introduce Kaluza-Klein ideas using these higher dimensions, but we do not do so in this paper.

We now conclude this section by enunciating the spin gauge principle, which has been used by several authors [1-4, 6-8, 10, 11]:

The Lagrangian density for a 2^n-component multispinor ψ is of the form

$$L_1=\tfrac{1}{2}[\bar{\psi}\mathrm{i}\Gamma_\mu(\partial^\mu\psi)-(\partial^\mu\psi)^+\Gamma\mathrm{i}\Gamma_\mu\psi]-\bar{\psi}M\psi+\bar{\psi}\mathrm{i}\Gamma_\mu\Omega^\mu\psi \tag{1.6a}$$

where $\{\Gamma_i, i=1,2,\ldots,2n\}$ is a set of generators of the Clifford algebra $C_{p,q}$ to which ψ belongs. The matrix M also belongs to $C_{p,q}$. The form of L_1 is determined by requiring local gauge invariance under the spin gauge transformations

$$\psi\to R\psi\equiv\psi' \qquad \Gamma_i\to R\Gamma_iR^{-1}\equiv\Gamma_i'. \tag{1.6b}$$

The generators of the transformation R are elements of the algebra $C_{p,q}$. We assume that the conjugation matrix Γ transforms to Γ', whose form can be deduced by demanding that $\Gamma_i'^+=\Gamma'\Gamma_i'\Gamma'^{-1}$; the transformation is then

$$\Gamma\to\Gamma'\equiv(R^{-1})^+\Gamma(R^{-1}) \tag{1.6c}$$

which ensures that

$$\bar{\psi}\to\bar{\psi}'\equiv\bar{\psi}R^{-1}. \tag{1.6d}$$

We shall use a set of generators $\{\Gamma_i\}$ which is spacetime dependent; that is, the elements Γ_i of the vector basis, whose inner products define the metric of a $2n$-dimensional flat space, are position dependent. The set $\{\Gamma_i\}$ is defined from the constant set $\{\Gamma_i^0\}$, which represents the same algebra, by a similarity transformation of the form

$$\Gamma_i=R\Gamma_i^0R^{-1}. \tag{1.7}$$

The similarity matrix R is the exponential of a Clifford number in the constant representation of $C_{p,q}$ with position dependent coefficients. The transformation (1.7) means that

$$\mathrm{Tr}\,\Gamma_i=\mathrm{Tr}\,\Gamma_i^0. \tag{1.8}$$

One advantage of spin gauge theories over standard theories is that there are more allowable transformations leaving a given Lagrangian invariant. In standard Yang-Mills gauge theories, the Γ_μ in (1.6a) are always taken as $I_k \otimes \gamma_\mu$ and the gauge transformations R as $S \otimes I_4$, where S is a $k \times k$ matrix. Thus, in Yang-Mills theories, R always commutes with Γ_μ and, in order that $\bar{\psi}$ transforms to $\bar{\psi} R^{-1}$, R must be unitary. Consequently, the range of Yang-Mills gauge transformations is restricted. However, in spin gauge theories, the invariance of L_1 can be retained even when R and Γ_i do not commute, and also when R is not unitary. If some of the elements Γ_A of the Clifford algebra used as generators of R do not commute with Γ_i, then, in order to ensure reality of the Lagrangian, it is only necessary that they are anti-self conjugate in the sense that

$$\Gamma_A = -\bar{\Gamma}_A$$

where the bar conjugate Γ_A is defined by

$$\bar{\Gamma}_A = \Gamma^{-1} \Gamma_A^+ \Gamma. \tag{1.9}$$

2. The lepton Lagrangian

We express our theory of the electron and its neutrino in terms of representations of the Clifford algebra $C_{2,6}$. The representations are defined in terms of a set of spacetime dependent generators of the Dirac algebra $\{\gamma_\mu(x);\ \mu, \nu = 1, 2, 3, 4\}$ and two sets of constant Pauli matrices $\{\lambda_i\}$ and $\{\rho_j\}$, where $i, j = 1, 2, 3$; we also take λ_4 and ρ_4 to be (2×2) unit matrices. The basis elements of the Clifford algebra are then

$$\lambda_i \rho_j \gamma_A (i, j = 1, 2, 3, 4;\ A = 1, 2, \ldots, 16). \tag{2.1}$$

The elements of the algebra are in general space dependent through the set $\{\gamma_A(x)\}$. However, to enable us to use the usual four-component left and right helicity projection operators $\frac{1}{2}(I \pm \mathrm{i}\gamma_5)$, we impose the condition that γ_5 is constant. Gauge variations of the helicity operators are possible, and have been investigated by Barut and McEwan [10].

The matrices (2.1) act upon 16-component spinors which, for leptons, have the form

$$\psi = (\varepsilon_L \varepsilon_R \nu_L \nu_R)^T \tag{2.2}$$

where $\varepsilon_{L,R}$ are the four-component left- and right-handed electron spinors

$$\varepsilon_L = \tfrac{1}{2}(I + \mathrm{i}\gamma_5)\varepsilon \qquad \varepsilon_R = \tfrac{1}{2}(I - \mathrm{i}\gamma_5)\varepsilon \tag{2.3a}$$

and likewise for the left- and right-handed neutrino spinors ν_L and ν_R. The conjugates of the spinors (2.3a) are defined so that the suffixes L, R indicate projection operators $\frac{1}{2}(I + \mathrm{i}\gamma_5)$ and $\frac{1}{2}(I - \mathrm{i}\gamma_5)$ respectively; thus

$$\bar{\varepsilon}_L = \tfrac{1}{2}\bar{\varepsilon}(I + \mathrm{i}\gamma_5) \qquad \bar{\varepsilon}_R = \tfrac{1}{2}\bar{\varepsilon}(I - \mathrm{i}\gamma_5) \tag{2.3b}$$

and likewise for the conjugate neutrino spinors. It is necessary for all of these spinors to have four components, in order to provide a sufficient number of dimensions to accommodate the space of SU(2) transformations and to allow transformation to the 'quark representation' at a later stage. If γ is the conjugation matrix for the particular representation of the Dirac algebra used, we choose the conjugation matrix Γ of the $C_{2,6}$ representation to be $\lambda_4 \rho_1 \gamma$, so that the conjugate to ψ is

$$\bar{\psi} = \psi^+ \lambda_4 \rho_1 \gamma = (\bar{\varepsilon}_L \bar{\varepsilon}_R \bar{\nu}_L \bar{\nu}_R). \tag{2.4}$$

This choice of Γ ensures that the norm of ψ is

$$\begin{aligned}\bar{\psi}\psi &= \bar{\varepsilon}_{\mathrm{L}}\varepsilon_{\mathrm{L}} + \bar{\varepsilon}_{\mathrm{R}}\varepsilon_{\mathrm{R}} + \bar{\nu}_{\mathrm{L}}\nu_{\mathrm{L}} + \bar{\nu}_{\mathrm{R}}\nu_{\mathrm{R}} \\ &= \bar{\varepsilon}\varepsilon + \bar{\nu}\nu\end{aligned} \tag{2.5}$$

the sum of the usual norms of the electron and neutrino spinors.

For the lepton representation, we use the $C_{2,6}$ basis of anti-commuting vectors

$$\begin{aligned}\Gamma_\mu &= \lambda_4\rho_1\gamma_\mu \qquad (\mu = 1, 2, 3, 4) \\ \Gamma_5 &= -\mathrm{i}\lambda_2\rho_2 I \\ \Gamma_6 &= \mathrm{i}\lambda_1\rho_2 I \\ \Gamma_7 &= \lambda_4\rho_1\gamma_5 \\ \Gamma_8 &= \lambda_3\rho_2 I.\end{aligned} \tag{2.6}$$

The metric (g_{ij}) of the 8-space is thus defined by

$$\begin{aligned}g_{ii}I_{16} &= \Gamma_i^2 = -I_{16} \qquad (i = 1, 2, 3, 5, 6, 7). \\ g_{44}I_{16} &= \Gamma_4^2 = g_{88}I_{16} = \Gamma_8^2 = +I_{16}.\end{aligned} \tag{2.7}$$

The x-dependent spacetime vector basis $\{\gamma_\mu\}$ is thus 'embedded' in the 8-space vector basis $\{\Gamma_1, \Gamma_2, \ldots, \Gamma_8\}$ through the first four components of (2.6).

The free Lagrangian density, in 16-component form, is

$$L_0 = \tfrac{1}{2}[\bar{\psi}\mathrm{i}\Gamma_\mu(\partial^\mu\psi) - (\partial^\mu\psi)^+\Gamma\mathrm{i}\Gamma_\mu\psi] - \tfrac{1}{2}m\bar{\psi}(\lambda_4 + \lambda_3)\rho_4\psi \tag{2.8}$$

the factor $\frac{1}{2}m(\lambda_4 + \lambda_3)$ provides a factor m for the electron spinors in the mass term, and a factor 0 for the neutrino spinors. The lepton kinetic terms are given by decomposing the first term in (2.8); for instance,

$$\begin{aligned}\bar{\psi}\mathrm{i}\lambda_4\rho_1\gamma_\mu(\partial^\mu\psi) &= [\bar{\varepsilon}_{\mathrm{L}}\bar{\varepsilon}_{\mathrm{R}}\bar{\nu}_{\mathrm{L}}\bar{\nu}_{\mathrm{R}}]\mathrm{i}\begin{bmatrix}0 & \gamma_\mu & 0 \\ \gamma_\mu & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \gamma_\mu\end{bmatrix}\begin{bmatrix}0 & \partial^\mu\varepsilon_{\mathrm{L}} \\ 0 & \partial^\mu\varepsilon_{\mathrm{R}} \\ \gamma_\mu & \partial^\mu\nu_{\mathrm{L}} \\ 0 & \partial^\mu\nu_{\mathrm{R}}\end{bmatrix} \\ &= \bar{\varepsilon}_{\mathrm{L}}\mathrm{i}\gamma_\mu\partial^\mu\varepsilon_{\mathrm{R}} + \bar{\varepsilon}_{\mathrm{R}}\mathrm{i}\gamma_\mu\partial^\mu\varepsilon_{\mathrm{L}} + \bar{\nu}_{\mathrm{L}}\mathrm{i}\gamma_\mu\partial^\mu\nu_{\mathrm{R}} + \bar{\nu}_{\mathrm{R}}\mathrm{i}\gamma_\mu\partial^\mu\nu_{\mathrm{L}} \\ &= \tfrac{1}{2}[\bar{\varepsilon}(I + \mathrm{i}\gamma_5)\mathrm{i}\gamma_\mu\partial^\mu\varepsilon + \bar{\varepsilon}(I - \mathrm{i}\gamma_5)\mathrm{i}\gamma_\mu\partial^\mu\varepsilon \\ &\quad + \bar{\nu}(I + \mathrm{i}\gamma_5)\mathrm{i}\gamma_\mu\partial^\mu\nu + \bar{\nu}(I - \mathrm{i}\gamma_5)\mathrm{i}\gamma_\mu\partial^\mu\nu] \\ &= \bar{\varepsilon}\mathrm{i}\gamma_\mu\partial^\mu\varepsilon + \bar{\nu}\mathrm{i}\gamma_\mu\partial^\mu\nu.\end{aligned}$$

The SU(2) symmetry group in the GSW theory is generated by the rotation operators in the space spanned by $\{\Gamma_5, \Gamma_6, \Gamma_7\}$; these are

$$U_1 = \tfrac{1}{2}\mathrm{i}\Gamma_6\Gamma_7 = \tfrac{1}{2}\mathrm{i}\lambda_1\rho_3\gamma_5 \tag{2.9a}$$

$$U_2 = \tfrac{1}{2}\mathrm{i}\Gamma_7\Gamma_5 = \tfrac{1}{2}\mathrm{i}\lambda_2\rho_3\gamma_5 \tag{2.9b}$$

$$U_3 = \tfrac{1}{2}\mathrm{i}\Gamma_5\Gamma_6 = \tfrac{1}{2}\lambda_3\rho_4 I. \tag{2.9c}$$

The generator of the U(1) group is taken to be

$$P = \tfrac{1}{2}\mathrm{i}\Gamma_5\Gamma_6\Gamma_7\Gamma_8 = \tfrac{1}{2}\mathrm{i}\lambda_4\rho_3\gamma_5 \tag{2.9d}$$

a multiple of the pseudoscalar of the 4-space spanned by $\{\Gamma_5, \Gamma_6, \Gamma_7, \Gamma_8\}$; this ensures that P commutes with $\{U_a; a=1,2,3\}$. Also, P and the set $\{U_a; a=1,2,3\}$ all commute with the 16×16 helicity projection operators

$$h_\pm = \tfrac{1}{2}(I_{16} \pm \mathrm{i}\lambda_4\rho_4\gamma_5). \tag{2.10}$$

The spin gauge transformation $R(x)$ which generates the GSW interaction for leptons is

$$R(x) = \exp[-\mathrm{i}gh_+U_a\theta_a(x) - \mathrm{i}g'(h_-U_3 + P)\theta_4(x)] \tag{2.11}$$

with summation over $a = 1, 2, 3$. The x-dependence of R is through the θ_a and θ_4 alone, since the elements of the Clifford algebra in (2.11) are constant. The free Lagrangian (2.8) is not invariant under the transformations (1.6). However, by taking the lepton Lagrangian to be

$$L_1 = \tfrac{1}{2}[\bar{\psi}\mathrm{i}\Gamma_\mu(D^\mu\psi) - \bar{\psi}\bar{D}^\mu\mathrm{i}\Gamma_\mu\psi] - \tfrac{1}{2}m\bar{\psi}(\lambda_4+\lambda_3)\rho_4\psi \tag{2.12}$$

where

$$D^\mu\psi = \partial^\mu\psi - \Omega^\mu\psi \qquad \bar{\psi}\bar{D}^\mu = (\partial^\mu\psi)^\dagger\Gamma - \bar{\psi}\bar{\Omega}^\mu \tag{2.13a}$$

and

$$\begin{aligned}\Omega^\mu &= \mathrm{i}gh_+U_aW_a^\mu + \mathrm{i}g'(h_-U_3+P)W_4^\mu \\ &= \tfrac{1}{2}\{\mathrm{i}gh_+(\mathrm{i}\lambda_j\rho_3\gamma_5W_j^\mu + \lambda_3\rho_4IW_3^\mu) \\ &\quad + \mathrm{i}g'(h_-\lambda_3\rho_4I + \mathrm{i}\lambda_4\rho_3\gamma_5)W_4^\mu\}\end{aligned} \tag{2.13b}$$

with summation over $j = 1, 2$, we can guarantee spin gauge invariance provided that

$$\Omega^\mu \to R\Omega^\mu R^{-1} + R(\partial^\mu R^{-1}). \tag{2.14a}$$

The requirement that γ_5, $\{\lambda_i\}$ and $\{\rho_j\}$ be constant, together with the identities $h_+h_- \equiv h_-h_+ \equiv 0$, ensure that the transformation (2.14a) splits into the following gauge transformations of the fields W_a^μ and W_4^μ:

$$h_+U_aW_a^\mu \to R_1h_+U_aW_a^\mu R_1^{-1} - \mathrm{i}g^{-1}R_1(\partial^\mu R_1^{-1}) \tag{2.14b}$$

$$W_4^\mu \to W_4^\mu + \partial^\mu\theta_4 \tag{2.14c}$$

where $R_1 = \exp[-\mathrm{i}gh_+U_a\theta_a(x)]$. The transformations (2.14b) and (2.14c) are the familiar GSW SU(2) and U(1) gauge transformations respectively.

The interaction terms in the Lagrangian (2.12) are

$$-\bar{\psi}\mathrm{i}\lambda_4\rho_1\gamma_\mu\Omega^\mu\psi. \tag{2.15}$$

We now reduce to four-component form the terms given by substitution of (2.13a) into (2.15). The terms involving the charged fields $\{W_j^\mu\}$ give

$$\begin{aligned}&-\tfrac{1}{2}\mathrm{i}g\bar{\psi}(\mathrm{i}\rho_1)\mathrm{i}(\lambda_1\not{W}_1 + \lambda_2\not{W}_2)\rho_3h_+\gamma_5\psi \\ &= -(\mathrm{i}g/\sqrt{2})[\bar{\varepsilon}_\mathrm{L}\bar{\varepsilon}_\mathrm{R}\bar{\nu}_\mathrm{L}\bar{\nu}_\mathrm{R}]\begin{bmatrix}0 & 0 & 0 & -\mathrm{i}\not{W}_- \\ 0 & 0 & \mathrm{i}\not{W}_- & 0 \\ 0 & -\mathrm{i}\not{W}_+ & 0 & 0 \\ \mathrm{i}\not{W}_+ & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\varepsilon_\mathrm{L} \\ 0 \\ \nu_\mathrm{L} \\ 0\end{bmatrix} \\ &= (g/\sqrt{2})(\bar{\varepsilon}_\mathrm{R}\not{W}_-\nu_\mathrm{L} + \bar{\nu}_\mathrm{R}\not{W}_+\varepsilon_\mathrm{L}).\end{aligned} \tag{2.16}$$

The remaining terms in (2.15) can be reduced in a similar way, giving

$$\tfrac{1}{2}g\bar{\psi}\lambda_3\rho_1 W\!\!\!/_3 h_+\psi = \tfrac{1}{2}g(\bar{\varepsilon}_R W\!\!\!/_3\varepsilon_L - \bar{\nu}_R W\!\!\!/_3\nu_L) \tag{2.17}$$

$$\tfrac{1}{2}g'\bar{\psi}\lambda_3\rho_1 W\!\!\!/_4 h_-\psi = \tfrac{1}{2}g'(\bar{\varepsilon}_L W\!\!\!/_4\varepsilon_R - \bar{\nu}_L W\!\!\!/_4\nu_R) \tag{2.18}$$

and

$$-\tfrac{1}{2}\mathrm{i}g'\bar{\psi}\lambda_4\rho_2 W\!\!\!/_4(\mathrm{i}\gamma_5)\psi = \tfrac{1}{2}g'(\bar{\varepsilon}W\!\!\!/_4\varepsilon + \bar{\nu}W\!\!\!/_4\nu). \tag{2.19}$$

The standard definition of the Weinberg angle θ_W gives

$$g = e/\sin\theta_W \qquad g' = e/\cos\theta_W. \tag{2.20}$$

where e is the charge on the electron. Then introducing the fields A^μ and Z^μ through

$$W_3^\mu = \sin\theta_W A^\mu - \cos\theta_W Z^\mu \tag{2.21a}$$

$$W_4^\mu = \cos\theta_W A^\mu + \sin\theta_W Z^\mu. \tag{2.21b}$$

The sum of the neutral field interaction terms (2.17), (2.18) and (2.19) reduce in the usual way to

$$e\bar{\varepsilon}A\!\!\!/\varepsilon + \tfrac{1}{2}g(2\sin^2\theta_W\bar{\varepsilon}Z\!\!\!/\varepsilon - \bar{\varepsilon}_R Z\!\!\!/\varepsilon_L + \bar{\upsilon}_R Z\!\!\!/\upsilon_L)/\cos\theta_W. \tag{2.22}$$

Since expressions (2.16) and (2.22) define the standard electroweak interactions of the leptons, we have shown that (2.13b) is the correct covariant derivative in this theoretical framework.

The form of the covariant derivative defined by (2.13) and (2.14) ensures that $D^\mu\psi$ is covariant under the transformation (2.11), that is,

$$D^\mu\psi \to RD^\mu\psi. \tag{2.23}$$

We can deduce that the commutator $[D^\mu, D^\nu]$ transforms by

$$[D^\mu, D^\nu] \to R[D^\mu, D^\nu]R^{-1} \tag{2.24}$$

and so can be used to construct the boson kinetic terms in the Lagrangians. Using the form of D^μ given in (2.13) and the constancy of the transformation generators, it is not difficult to show that

$$\begin{aligned}[D^\mu, D^\nu] = {} & -\mathrm{i}gh_+U_a(\partial^\mu W_a^\nu - \partial^\nu W_a^\mu + g\varepsilon_{abc}[W_b^\mu, W_c^\nu]) \\ & -\mathrm{i}g'(h_-U_3 + P)(\partial^\mu W_4^\nu - \partial^\nu W_\mu^4).\end{aligned} \tag{2.25}$$

From this expression we can project out the curls of the four boson fields as gauge-invariant traces:

$$\mathrm{i}\,\mathrm{Tr}\{h_+U_a[D^\mu, D^\nu]\}/16g = \tfrac{1}{2}(\partial^\mu W_a^\nu - \partial^\nu W_a^\mu + g\varepsilon_{abc}[W_b^\mu, W_c^\nu])$$

$$\mathrm{i}\,\mathrm{Tr}\{(h_-U_3 + P)[D^\mu, D^\nu]\}/48g' = \tfrac{1}{2}(\partial^\mu W_4^\nu - \partial^\nu W_4^\mu).$$

From these expressions we can construct the Lagrangian terms for the free bosons:

$$\begin{aligned}(-\tfrac{1}{4})[(\partial_\mu W_{a\nu} - \partial_\nu W_{a\mu} + g\varepsilon_{abc}[W_{b\mu}, W_{c\nu}])(\partial^\mu W_a^\nu - \partial^\nu W_a^\mu \\ + g\varepsilon_{abc}[W_b^\mu, W_c^\nu]) + (\partial_\mu W_{4\nu} - \partial_\nu W_{4\mu})(\partial^\mu W_4^\nu - \partial^\nu W_4^\mu)].\end{aligned} \tag{2.26}$$

3. Mass and the extended covariant derivative

The set of vectors $\{\Gamma^i(x)\}$ is in general x-dependent, but can be related to a constant set $\{\Gamma_0^i\}$ by a gauge transformation $R(x)$:

$$\Gamma^i(x) = R\Gamma_0^i R^{-1}. \tag{3.1}$$

Dongpei [2] has pointed out that choosing a constant set, as we do in practical calculations, for instance, in the decomposition of (2.15), is thus equivalent to fixing the gauge, apart from a constant generalised Lorentz transformation. We call this the 'Dongpei gauge'. He has also shown that extra gauge-invariant terms can be introduced into the Lagrangian; these become boson mass terms when $\Gamma^i(x) = \Gamma_0^i$. By choosing the Dongpei gauge, the set of spacetime vectors $\{\gamma^\mu\}$ representing the frame field becomes constant. In this section we shall show how boson mass terms arise naturally if we treat the frame field like other fields by introducing it into the covariant derivative.

First, we must recognise that $\gamma^\mu(x)$ is dimensionless, and needs to be multiplied by a constant of the dimension of (mass/coupling constant) to ensure that it has the same dimensions as the W fields; for classical fields it is not correct to treat coupling constants as dimensionless. Introducing the universal constant η, we amend the definition of the 'frame field' to include this factor:

$$\phi^\mu(x) = \eta\gamma^\mu(x). \tag{3.2}$$

To see how the frame field can be incorporated into the covariant derivative, consider first the free Dirac equation

$$(\mathrm{i}\gamma_\mu\partial^\mu - Im)\psi = 0.$$

This can be written as

$$\mathrm{i}\gamma_\mu(I\partial^\mu + \tfrac{1}{4}im\gamma^\mu)\psi = 0.$$

If we now define a coupling constant f by

$$m = 4f\eta \tag{3.3}$$

this equation takes the form

$$\mathrm{i}\gamma_\mu(I\partial^\mu + if\phi^\mu)\psi = 0.$$

The term in brackets in this equation has the form of a covariant derivative; the constant f, proportional to the mass, is the strength of coupling of the fermion to the frame field. We therefore see mass as a form of 'drag' or friction exerted by the frame field upon the fermion. A massless particle moves without friction through the frame field, at the velocity of light. We call the constant η the 'frame inertia', since it causes particles to travel more slowly than their 'natural speed', the speed of light.

Let us now apply this idea to the free Lagrangian of the leptons in our model. The mass m of the electron can be included if we modify the Lagrangian (2.8) to

$$L_0 = \tfrac{1}{2}\bar{\psi}[\mathrm{i}\lambda_4\rho_1\gamma_\mu(\partial^\mu\psi) - \tfrac{1}{2}(\lambda_4 + \lambda_3)\rho_4 Im\psi] + \mathrm{conj} \tag{3.4}$$

the factor $(\lambda_4 + \lambda_3)$ ensuring that the neutrino mass is zero. Factoring $\mathrm{i}\lambda_4\rho_1\gamma_\mu$ from the terms in square brackets, the derivative acting on ψ is augmented to

$$\lambda_4\rho_4 I\partial^\mu + \mathrm{i}(\lambda_4 + \lambda_3)\rho_1 m\gamma^\mu/8 = I_{16}\partial^\mu + \tfrac{1}{2}\mathrm{i}(\lambda_4 + \lambda_3)\rho_1 f\phi^\mu. \tag{3.5}$$

If we now include the term $-\Omega^{\mu}$, as in (2.13), we obtain the 'extended covariant derivative'

$$\Delta^{\mu} = I_{16}\partial^{\mu} - \Omega^{\mu} + \tfrac{1}{2}\mathrm{i}(\lambda_4 + \lambda_3)\rho_1 f\phi^{\mu}$$
$$= I_{16}\partial^{\mu} - \tfrac{1}{2}\{\mathrm{i}gh_{+}(\mathrm{i}\lambda_j\rho_3\gamma_5 W_j^{\mu} + \lambda_3\rho_4 I W_3^{\mu})$$
$$+\mathrm{i}g'(h_{-}\lambda_3\rho_4 I + \mathrm{i}\lambda_4\rho_3\gamma_5)W_4^{\mu}\} + \tfrac{1}{2}\mathrm{i}(\lambda_4 + \lambda_3)\rho_1 f\phi^{\mu}. \tag{3.6}$$

The last term in (3.6) is simply a sum of elements of the algebra, all of which transform as the basis vectors do; thus it transforms by (1.6*b*) and by using (2.23), we deduce that

$$\Delta^{\mu}\psi \rightarrow R\Delta^{\mu}\psi$$

where R is still defined by (2.11), so that the GSW interactions are preserved. Although the reason for the covariance of the last term in Δ^{μ} is exceptional, we see that the frame field appears in the extended covariant derivative in exactly the same form as the W fields. This is the reason for treating the mass term in the way we have. However, in this paper, the inclusion of the last term in (3.6) does not correspond to an additional symmetry transformation. We shall comment on this at the end of the paper.

The analysis of § 2 has shown that the Lagrangian

$$L = \tfrac{1}{2}[\bar{\psi}\mathrm{i}\Gamma_{\mu}\Delta^{\mu}\psi - (\Delta^{\mu}\psi)^{+}\Gamma\mathrm{i}\Gamma_{\mu}\psi] \tag{3.7}$$

formed from (2.8) by replacing ∂^{μ} by Δ^{μ}, gives the correct electroweak interactions for leptons; it also contains the electron mass term, with the neutrino mass equal to zero. In order to derive the free Lagrangian terms for the boson fields, we consider the commutator $[\Delta^{\mu}, \Delta^{\nu}]$ of the extended covariant derivatives. This commutator is the sum of three different types of term, which are denoted by A, B, C.

A terms. These are the terms already discussed in § 2, arising from the derivative terms and the W field terms. As in (2.25), they are

$$M_{\mathrm{A}}^{\mu\nu} = -\mathrm{i}gh_{+}U_a(\partial^{\mu}W_a^{\nu} - \partial^{\nu}W_a^{\mu} + g\varepsilon_{abc}[W_b^{\mu}, W_c^{\nu}])$$
$$-\mathrm{i}g'(h_{+}U_3 + P)(\partial^{\mu}W_4^{\nu} - \partial^{\nu}W_4^{\mu}). \tag{3.8}$$

B terms. These are the terms arising from the commutator of the ϕ field and W field terms. They are

$$M_{\mathrm{B}}^{\mu\nu} = \tfrac{1}{4}fg[(\lambda_3 + \lambda_4)\rho_1\phi^{\mu}, (I + \mathrm{i}\gamma_5)U_a W_a^{\nu}]$$
$$+\tfrac{1}{4}fg'[(\lambda_3 + \lambda_4)\rho_1\phi^{\mu}, \{(I - \mathrm{i}\gamma_5)U_3 + 2P\}W_4^{\nu}] - (\mu \rightleftarrows \nu \text{ terms}). \tag{3.9}$$

C terms. These are the terms arising from the derivative and the ϕ-field terms. They are

$$M_{\mathrm{C}}^{\mu\nu} = \tfrac{1}{2}\mathrm{i}f(\lambda_3 + \lambda_4)\rho_1(\partial^{\mu}\phi^{\nu} - \partial^{\nu}\phi^{\mu}) - \tfrac{1}{4}\{f(\lambda_3 + \lambda_4)\rho_1\}^2[\phi^{\mu}, \phi^{\nu}]. \tag{3.10}$$

We shall discuss these terms in reverse order.

For a general gauge the first of the C terms (3.10) contains the curl of the frame field. However, we shall take the matrices to be constant by choosing the Dongpei gauge; then the curl vanishes as a classical field and the second term is equal to

$$-f^2\eta^2(\lambda_4 + \lambda_3)\rho_4\gamma^{\mu}\gamma^{\nu} \qquad (\mu \neq \nu). \tag{3.11}$$

The B terms, by given (3.9), are of the same general form as those used by Dongpei to introduce boson masses into the Lagrangian. We shall see that these terms do in fact provide exactly the correct mass terms for the GSW bosons, provided that the constant f is suitably chosen. By using a covariant derivative containing the frame field, these boson mass terms arise as multiples of the electron mass; in more general models, such as the model we shall consider in § 4, the extended covariant derivative would contain more fermion masses, and a set of relationships between fermion and boson masses would result. The frame field concept provides a systematic scheme for introducing Dongpei-type mass terms through the extended covariant derivative.

We shall now analyse the B terms in some detail, since the details are both instructive and important. We are using Dongpei gauge, so $f\phi^{\mu}$ is constant and equal to $\frac{1}{4}m\gamma^{\mu}$. Since P commutes with $(\lambda_3+\lambda_4)\rho_1\phi^{\mu}$, it does not contribute to (3.9). Using the values of $\{U_a\}$ given by (2.9), the remaining terms give both charged and neutral field contributions. The charged field contributions arise from the terms in the first term of (3.9) with $a=1, 2$, and are equal to

$$\begin{aligned}(mg/32)&[(\lambda_3+\lambda_4)\rho_1\gamma^{\mu}, (I+\mathrm{i}\gamma_5)\mathrm{i}\lambda_i\rho_3\gamma_5]W_i^{\nu}-(\mu\rightleftarrows\nu\text{ terms})\\ &=(-\mathrm{i}mg/16)[\lambda_i\rho_2\gamma^{\mu}-\varepsilon_{3ij}\lambda_j\rho_2\gamma^{\mu}\gamma_5]W_i^{\nu}-(\mu\rightleftarrows\nu\text{ terms})\end{aligned}\tag{3.12a}$$

where i and j are summed over 1 and 2, since one λ_3 and one λ_4 term vanish. The neutral field contributions arise from the first term of (3.9) with $a=3$, and from the second term, with P omitted. The $\mathrm{I}U_3$ terms are zero, but the $\mathrm{i}\gamma_5U_3$ terms contribute

$$\begin{aligned}(mg/32)&[(\lambda_3+\lambda_4)\rho_1\gamma^{\mu}, \mathrm{i}\lambda_3\rho_4\gamma_5]W_3^{\nu}\\ &+(mg'/32)[(\lambda_3+\lambda_4)\rho_1\gamma^{\mu}, -\mathrm{i}\lambda_3\rho_4\gamma_5]W_4^{\nu}-(\mu\rightleftarrows\nu\text{ terms})\\ &=(\mathrm{i}m/16)(\lambda_3+\lambda_4)\rho_1\gamma^{\mu}\gamma_5(gW_3^{\nu}-g'W_4^{\nu})-(\mu\rightleftarrows\nu\text{ terms}).\end{aligned}\tag{3.12b}$$

The opposite signs arising from the helicity factors are extremely important, since by (2.21) and (2.20) this expression is just

$$-(\mathrm{i}mg/16)(\lambda_3+\lambda_4)\rho_1\gamma^{\mu}\gamma_5Z^{\nu}\sec\theta_W-(\mu\rightleftarrows\nu\text{ terms})\tag{3.12c}$$

with zero contribution from the field A^{ν}. The B terms (3.9) are equal to the sum of of (3.12a) and (3.12c), giving

$$\begin{aligned}M_{\mathrm{B}}^{\mu\nu}=(-\mathrm{i}mg/16)\{&[\lambda_i\rho_2\gamma^{\mu}-\varepsilon_{3ij}\lambda_j\rho_2\gamma^{\mu}\gamma_5]W_i^{\nu}\\ &+(\lambda_3+\lambda_4)\rho_1\gamma^{\mu}\gamma_5Z^{\nu}\sec\theta_W\}-(\mu\rightleftarrows\nu\text{ terms}).\end{aligned}\tag{3.13}$$

As discussed in § 2, the expression $M_{\mathrm{A}}^{\mu\nu}$ in (3.8) is gauge covariant. The sum $M_{\mathrm{B}}^{\mu\nu}+M_{\mathrm{C}}^{\mu\nu}$ is also gauge covariant. From this gauge covariant expression, we can form invariant mass terms by generalising Dongpei's method. We define from (3.10) and (3.13) the gauge and relativistically invariant element

$$\begin{aligned}\{\mathrm{Tr}[g_{\mu\rho}g_{\nu\sigma}&(M_{\mathrm{B}}^{\mu\nu}+M_{\mathrm{C}}^{\mu\nu})(M_{\mathrm{B}}^{\rho\sigma}+M_{\mathrm{C}}^{\rho\sigma}]\}/16\\ &=\{\mathrm{Tr}[g_{\mu\rho}g_{\nu\sigma}M_{\mathrm{B}}^{\mu\nu}M_{\mathrm{B}}^{\rho\sigma}]+\mathrm{Tr}[g_{\mu\rho}g_{\nu\sigma}M_{\mathrm{C}}^{\mu\nu}M_{\mathrm{C}}^{\rho\sigma}]\}/16.\end{aligned}\tag{3.14}$$

By using (1.8), the second trace term in (3.14) becomes

$$(-f^2/8)g_{\mu\rho}g_{\nu\sigma}\,\mathrm{Tr}\{(\partial^{\mu}\phi^{\nu}-\partial^{\nu}\phi^{\mu})(\partial^{\rho}\phi^{\sigma}-\partial^{\sigma}\phi^{\rho})-f^2[\phi^{\mu}, \phi^{\nu}][\phi^{\rho}, \phi^{\sigma}]\}.\tag{3.15}$$

Apart from the trace operation and a factor of $\frac{1}{2}f^2$, the first term in (3.15) is the usual Lagrangian for a vector field. Although this first term is zero in the Dongpei gauge, it is reasonable to normalise (3.15) by dividing by $\frac{1}{2}f^2$. Using this normalisation in the Dongpei gauge, (3.15) becomes

$$-24f^2\eta^4$$

which is constant and changes the zero of the kinetic energy.

The first trace term in (3.14) contains a factor g^2, and must be divided by the square of a coupling constant to produce a boson Lagrangian mass term. We must use the same normalisation for the B terms as for the C terms in (3.14). Thus dividing by $\frac{1}{2}f^2$, we obtain

$$(1/8f^2)\,\mathrm{Tr}[g_{\mu\rho}g_{\nu\sigma}M_{\mathrm{B}}^{\mu\nu}M_{\mathrm{B}}^{\rho\sigma}]. \tag{3.16}$$

Using (3.13) and (3.14) and trace formulae such as

$$\mathrm{Tr}[\lambda_2^2\rho_2^2\gamma_\mu\gamma^\mu] = 4\times 16 = 64 \tag{3.17a}$$

$$\mathrm{Tr}[\lambda_1^2\rho_2^2\gamma_\mu\gamma_5\gamma^\mu\gamma_5] = 4\times 16 = 64 \tag{3.17b}$$

and

$$\mathrm{Tr}[(\lambda_3+\lambda_4)^2\rho_1^2\gamma_\mu\gamma_5\gamma^\mu\gamma_5] = 4\times 2\times 16 = 128 \tag{3.17c}$$

the expression (3.16) reduces in the Dongpei gauge to

$$(1/4f^2)(img/16)^2 128[W_{i\nu}W_i^\nu + \sec^2\theta_{\mathrm{W}} Z_\nu Z^\nu]$$
$$= -\tfrac{1}{2}(mg/2f)^2[W_{i\nu}W_i^\nu + \sec^2\theta_{\mathrm{W}} Z_\nu Z^\nu]. \tag{3.18}$$

This choice of normalisation leads us to identify the mass of the W bosons in this model as

$$M_{\mathrm{W}} = mg/2f \tag{3.19}$$

we stress that this relationship is model dependent, since the electron mass m is the only fermion mass entering the model. A more complete theory would introduce higher fermion masses, such as that of the muon or the tau lepton or the quarks, which might be expected to alter the relation (3.19). In the next section, however, we shall see that this relationship is in a sense unaltered by the introduction of other fermions, even though the value of f is changed.

The crucially important facts about expression (3.18) are:

(*a*) these invariant terms are of the form of Lagrangian mass terms;

(*b*) the photon mass is zero;

(*c*) the mass ratio

$$M_Z/M_{\mathrm{W}} = \sec\theta_{\mathrm{W}} \tag{3.20}$$

independent of the choice of f or of the normalisation of the terms. Also, as we have already noted, opposite helicities are needed to give zero photon mass. We have shown therefore that, by choosing the matrix elements in this model to agree with the GSW theory, the extended covariant derivative (incorporating the frame field) gives the correct boson mass ratios in the Dongpei gauge. Our form of spin gauge theory thus provides an explanation of boson masses which does not depend upon the Higgs–Kibble mechanism.

Before discussing the A terms, it is worth noting certain points about the calculation of the B terms. First, we note that the W_i^ν, $i=1,2$, terms in (3.12*a*) are of a different form to the W_3^ν terms in (3.12*b*); but when the trace calculations are carried out, these two sets of terms contribute equal factors (128) through the sum of (3.17*a*) and (3.17*b*) and through (3.17*c*). So our procedure for introducing mass does not in the end violate SU(2) group covariance. What is remarkable is that, because of the $(\lambda_3+\lambda_4)$ factor in the frame field factors in (3.9), the helicity factors in the W field terms are essential to the group symmetry being reflected in the mass terms. Further, the equality of

(3.17*c*) to the sum of (3.17*a*) and (3.17*b*) is essential to establishing the mass relationship (3.20). We are of the opinion that the crucial part played by the helicity factors in this calculation indicates that these factors are not just some eccentricity of nature, but will prove to be an essential part of a full theory.

Now that we have used the B terms to obtain the Lagrangian mass terms, let us return to consideration of the A terms, which provide the 'kinetic' terms. Since $M_{\mathrm{A}}^{\mu\nu}$ is gauge covariant, the trace term

$$\{\mathrm{Tr}[g_{\mu\rho}g_{\nu\sigma}M_{\mathrm{A}}^{\mu\nu}M_{\mathrm{A}}^{\rho\sigma}]\}$$

is gauge invariant, independently of (3.14), and hence it can be normalised independently of (3.14). However, since both mass and kinetic terms arise from the commutator $[\Delta^{\mu}, \Delta^{\nu}]$ of the extended covariant derivative, it would be particularly neat if their normalisation factors were the same; then the full boson Lagrangian would be of the simple form

$$-K\ \mathrm{Tr}\{g_{\mu\rho}g_{\nu\sigma}[\Delta^{\mu}, \Delta^{\nu}][\Delta^{\rho}, \Delta^{\sigma}]\} \tag{3.21}$$

where K is an appropriate constant, such as $(1/8f^2)$. We shall discuss this possiblity further in § 5; for the present we note that, *in this model*, this simple procedure does not work. The fact that M_W is very much larger than m, the only fermion mass in the model, shows, through (3.19), that f (or whatever coupling constant we use to normalise the mass terms) is much smaller than g and g'; to produce independent kinetic terms for the W_a^{ν} $(a=1,2,3)$ fields and for the W_4^{ν} field, we have to follow the procedure outlined in § 2. It is interesting to note, however, that if g and g' satisfied the relation

$$g/g'=\sqrt{3} \tag{3.22}$$

we could then evaluate the kinetic terms together by calculating

$$(1/16g^2)\ \mathrm{Tr}[g_{\mu\rho}g_{\nu\sigma}M_{\mathrm{A}}^{\mu\nu}M_{\mathrm{A}}^{\rho\sigma}]. \tag{3.23}$$

Substituting from (3.8), and using trace formulae such as

$$\mathrm{Tr}[h_+U_1h_+U_1]=\mathrm{Tr}[h_+\lambda_4\rho_4]=8$$

and

$$\begin{aligned}\mathrm{Tr}[(h_+U_3+P)^2]&=\mathrm{Tr}[h_+U_3h_+U_3]+\mathrm{Tr}[\lambda_4\rho_4 I]\\&=8+16=24\end{aligned}$$

we find that (3.23) reduces to

$$\begin{aligned}-\tfrac{1}{4}[(\partial_{\mu}W_{a\nu}-\partial_{\nu}W_{a\mu}+g\varepsilon_{abc}[W_{b\mu}, W_{c\nu}])(\partial^{\mu}W_a^{\nu}-\partial^{\nu}W_a^{\mu}\\+g\varepsilon_{abc}[W_b^{\mu}, W_c^{\nu}])+(\partial_{\mu}W_{4\nu}-\partial_{\nu}W_{4\mu})(\partial^{\mu}W_4^{\nu}-\partial^{\nu}W_4^{\mu})]\end{aligned} \tag{3.24}$$

provided that (3.22) holds. This would imply that the Weinberg angle was 30°, so that $\sin^2\theta_W=0.25$. The most recent experimental results for this quantity place its value between 0.22 and 0.23. The model we are presenting only deals with the leptons of the first generation, and does not attempt to deal with strong interactions; within the limits of this model, the ratio (3.22) could be a good first approximation to the measured value. Then the kinetic terms in the Lagrangian, like the mass and frame field terms, can have a common normalisation factor. It could be that, in a full theory, the full boson Lagrangian is given by (3.21), implying certain relationships between fermion and boson masses: the introduction of large fermion masses (perhaps an especially large quark mass) into the extended covariant derivative would imply frame field couplings much larger than f, given by a variant of (3.3); this might allow a common normalisation of all boson Lagrangian terms.

4. The first generation quark Lagrangian

In two previous papers [4, 7] on spin gauge theories, we obtained the covariant derivatives for different particles from the lepton covariant derivative by transforming the representation of the Clifford algebra, leaving the form of the spinor unchanged. We adopt the same principle here and define the first generation quark representation by transforming the covariant derivatives of §§ 2 and 3. The transformation corresponds to a change of the eighth basis vector in the representation of $C_{2,6}$ used in § 2. The pseudoscalar $i\lambda_4\rho_3 I$ of the algebra $C_{2,6}$ can be regarded as a ninth basis vector Γ_9 in a representation of $C_{2,7}$; then the transformation T_α given by

$$\begin{aligned} T_\alpha &= \exp[i\Gamma_8\Gamma_9\alpha] \\ &= \exp[-i\lambda_3\rho_1 I\alpha] \end{aligned} \tag{4.1a}$$

with α constant, is a rotation in the 8-9 space.

Any element Γ_A of the algebra in the covariant derivative is transformed by the similarity transformation

$$T_\alpha \Gamma_A T_\alpha^{-1}. \tag{4.1b}$$

By transforming the elements of the lepton covariant derivative (2.13), we obtain the covariant derivative for a 16-component spinor ψ_α. We assume that the spinor ψ_α and its conjugate $\bar{\psi}_\alpha$ have the same form as ψ and $\bar{\psi}$ given by (2.2) and (2.4), that is,

$$\begin{aligned} \psi_\alpha &= (a_L a_R b_L b_R)^T \\ \bar{\psi}_\alpha &= (\bar{a}_L \bar{a}_R \bar{b}_L \bar{b}_R) \end{aligned}$$

for some four-component spinors a and b. As we pointed out in [4], we could replace the transformation (4.1b) of the elements in the covariant derivative by an equivalent transformation of the spinor of the form

$$\psi \to T_\alpha^{-1}\psi.$$

Since $\{\Gamma_\mu\}$ and Γ are invariant under the change of representation (4.1), the kinetic energy for the spinor ψ_α has the same form as the lepton kinetic energy. Of the generators h_+U_a $(a = 1, 2, 3)$, h_-U_3 and P used in the gauge transformation (2.11), and hence in (2.13), only P undergoes a change of representation under (4.1). The element P transforms to P' where

$$\begin{aligned} P' &= (I_{16}\cos 2\alpha - i\lambda_3\rho_1 I \sin 2\alpha)P \\ &= P\cos 2\alpha - i\lambda_3\rho_2\gamma_5 \sin 2\alpha. \end{aligned} \tag{4.2}$$

Hence the interaction terms for the a and b spinors corresponding to the lepton terms (2.16)-(2.18) are unchanged in form. However, the term (2.19) becomes

$$-ig'\bar{\psi}_\alpha(\lambda_4\rho_2\cos 2\alpha + \lambda_3\rho_3 \sin 2\alpha)\not{W}_4(i\gamma_5)\psi_\alpha. \tag{4.3}$$

The $\sin 2\alpha$ term in (4.3) is zero on decomposition to four-component spinors a and b; the vanishing occurs since in each term of the decomposition a four-component spinor and its conjugate of the same handedness are present. By analogy with (2.19), the non-zero term in (4.3) decomposes to

$$\tfrac{1}{2}g'\cos 2\alpha(\bar{a}\not{W}_4 a + \bar{b}\not{W}_4 b).$$

By introducing the fields A^μ and Z^μ through (2.21), the total neutral field interaction terms for the a and b spinors becomes

$$\tfrac{1}{2}e[\bar{a}\not{A}(\cos 2\alpha+1)a+\bar{b}\not{A}(\cos 2\alpha-1)b]+\tfrac{1}{2}g[\bar{a}\sin^2\theta_W(\cos 2\alpha+1)\not{Z}a-\bar{a}_R\not{Z}a_L$$
$$+\bar{b}\sin^2\theta_W(\cos 2\alpha-1)\not{Z}b+\bar{b}_R\not{Z}b_L]/\cos\theta_W. \tag{4.4}$$

For a given angle α, we wish to identify the spinors a and b with the down and up quark spinors d and u respectively. As we noted above, for any angle α, the kinetic energy terms and interaction terms with the charged W fields for the a and b spinors are identical to those of the down and up quarks. Therefore, we must choose α in order that the terms (4.4) give the correct neutral field interactions for the quarks. Taking

$$\cos 2\alpha=-\tfrac{1}{3} \tag{4.5}$$

$a=d$ and $b=u$, the terms (4.4) become

$$\tfrac{1}{3}e\bar{d}\not{A}d-\tfrac{2}{3}e\bar{u}\not{A}u+\tfrac{1}{2}g[\tfrac{2}{3}\sin^2\theta_W\bar{d}\not{Z}d-\bar{d}_R\not{Z}d_L-\tfrac{4}{3}\sin^2\theta_W\bar{u}\not{Z}u+\bar{u}_R\not{Z}u_L]/\cos\theta_W \tag{4.6}$$

which are the correct GSW interactions of the up and down quarks with the photon and Z-fields.

By changing the representation of the Clifford algebra in the lepton model of § 2 using the transformation (4.1) with $2\alpha=\cos^{-1}(-\tfrac{1}{3})$, we obtain the weak interaction Lagrangian for the first generation quarks. We note that this value of 2α is the interior angle of a regular tetrahedron. This suggests that the inclusion of quarks of three colours might lead to some form of tetrahedral symmetry between the leptons and the quarks [4].

If we apply the transformation (4.1) to the extended covariant derivative (3.6), we find that the lepton mass term

$$\tfrac{1}{2}[\bar{\psi}\tfrac{1}{2}(-\lambda_4-\lambda_3)\rho_4\,\mathrm{Im}\,\psi+\mathrm{conj}]$$

is unchanged when we transform to the quark representation. This would imply that the down quark had the same mass as the electron and that the up quark had zero mass. Thus the simple mass model of § 3 must be modified so that on transformation we generate the correct form of mass terms for the quarks. The modification is carried out by introducing into the lepton model extra mass terms [7], which are not invariant under the transformation (4.1).

We propose replacing the original mass terms in (3.7) by

$$\tfrac{1}{2}\bar{\psi}(-\mu_1\lambda_4-\mu_2\lambda_3)\rho_4 I\psi+\mathrm{conj} \tag{4.7a}$$

and adding the terms

$$\tfrac{1}{2}\bar{\psi}\mathrm{i}(-\mu_3\lambda_4-\mu_4\lambda_3)\rho_3\gamma_5\psi+\mathrm{conj}. \tag{4.7b}$$

By adding (4.7a) and (4.7b) we obtain the complete lepton mass term

$$\tfrac{1}{2}\bar{\psi}\mathrm{i}\lambda_4\rho_1\gamma_\mu[\mathrm{i}f_1\lambda_4\rho_1\phi^\mu+\mathrm{i}f_2\lambda_3\rho_1\phi^\mu-\mathrm{i}f_3\lambda_4\rho_2\gamma_5\phi^\mu-\mathrm{i}f_4\lambda_3\rho_2\gamma_5\phi^\mu]\psi+\mathrm{conj} \tag{4.8}$$

where

$$\mu_i=4f_i\eta \qquad i=1,2,3,4. \tag{4.9}$$

The extended covariant derivative is thus

$$\Delta^\mu=I_{16}\partial^\mu-\Omega^\mu+\mathrm{i}f_1\lambda_4\rho_1\phi^\mu+\mathrm{i}f_2\lambda_3\rho_1\phi^\mu-\mathrm{i}f_3\lambda_4\rho_2\gamma_5\phi^\mu-\mathrm{i}f_4\lambda_3\rho_2\gamma_5\phi^\mu. \tag{4.10}$$

On decomposition the terms (4.8) become in the Dongpei gauge

$$-\bar{\varepsilon}(\mu_1+\mu_2+\mu_3+\mu_4)\varepsilon-\bar{\nu}(\mu_1-\mu_2+\mu_3-\mu_4)\nu$$

and hence we require

$$\mu_1+\mu_2+\mu_3+\mu_4=m_e \tag{4.11a}$$

$$\mu_1-\mu_2+\mu_3-\mu_4=0 \tag{4.11b}$$

where m_e is the electron mass.

The commutator of the new extended covariant derivative (4.10) produces three different types of terms as in § 3. The A terms are the same as (3.8), and the B and C terms become respectively

$$\begin{aligned}M_B^{\mu\nu}=\tfrac{1}{2}g[(f_1\lambda_4\rho_1I+f_2\lambda_3\rho_1I-f_3\lambda_4\rho_2\gamma_5-f_4\lambda_3\rho_2\gamma_5)\phi^\mu,(I+\mathrm{i}\gamma_5)U_aW_a^\nu]\\+\tfrac{1}{2}g'[(f_1\lambda_4\rho_1I+f_2\lambda_3\rho_1I-f_3\lambda_4\rho_2\gamma_5-f_4\lambda_3\rho_2\gamma_5)\phi^\mu,\\\{(I-\mathrm{i}\gamma_5)U_3+2P\}W_4^\nu]-(\mu\rightleftarrows\nu\text{ terms})\end{aligned} \tag{4.12}$$

and

$$\begin{aligned}M_C^{\mu\nu}=\mathrm{i}(f_1\lambda_4\rho_1I+f_2\lambda_3\rho_1I-f_3\lambda_4\rho_2\gamma_5-f_4\lambda_3\rho_2\gamma_5)(\partial^\mu\phi^\nu-\partial^\nu\phi^\mu)\\-[(f_1\lambda_4\rho_1I+f_2\lambda_3\rho_1I-f_3\lambda_4\rho_2\gamma_5-f_4\lambda_3\rho_2\gamma_5)\phi^\mu,\\(f_1\lambda_4\rho_1I+f_2\lambda_3\rho_1I-f_3\lambda_4\rho_2\gamma_5-f_4\lambda_3\rho_2\gamma_5)\phi^\nu].\end{aligned} \tag{4.13}$$

The analysis of the B terms in the model of § 3 carries forward to our new model, with the opposite signs in the helicity factors playing a crucial role as before to ensure that the A^ν field gives a zero contribution. By analogy with (3.13), we find that the terms (4.12) become

$$\begin{aligned}M_B^{\mu\nu}=(-\mathrm{i}g/8)\{[\mu_1\lambda_i\rho_2\gamma^\mu-\mu_2\varepsilon_{3ij}\lambda_j\rho_2\gamma^\mu\gamma_5+\mu_3\lambda_i\rho_1\gamma^\mu\gamma_5-\mu_4\varepsilon_{3ij}\lambda_j\rho_1\gamma^\mu]W_i^\nu\\+[(\mu_1\lambda_3+\mu_2\lambda_4)\rho_1\gamma^\mu\gamma_5-(\mu_3\lambda_3+\mu_4\lambda_4)\rho_2\gamma^\mu]Z^\nu\sec\theta_W\}\\-(\mu\rightleftarrows\nu\text{ terms}).\end{aligned} \tag{4.14}$$

As in § 3, we define the gauge and relativistically invariant quantity

$$\{\mathrm{Tr}[g_{\mu\rho}g_{\nu\sigma}M_B^{\mu\nu}M_B^{\rho\sigma}]+\mathrm{Tr}[g_{\mu\rho}g_{\nu\sigma}M_C^{\mu\nu}M_C^{\rho\sigma}]\}/16.$$

The second trace term is

$$-\tfrac{1}{4}F^2g_{\mu\rho}g_{\nu\sigma}\,\mathrm{Tr}\{(\partial^\mu\phi^\nu-\partial^\nu\phi^\mu)(\partial^\rho\phi^\sigma-\partial^\sigma\phi^\rho)-F^2[\phi^\mu,\phi^\nu][\phi^\rho,\phi^\sigma]\} \tag{4.15}$$

where

$$F^2=f_1^2+f_2^2+f_3^2+f_4^2. \tag{4.16}$$

By analogy with (3.15), we normalise (4.15) by dividing by F^2. Using the same normalisation for the B terms, we obtain

$$\{\mathrm{Tr}[g_{\mu\rho}g_{\nu\sigma}M_B^{\mu\nu}M_B^{\rho\sigma}]\}/16F^2=-\tfrac{1}{2}(Mg/2F)^2[W_{i\nu}W_i^\nu+\sec^2\theta_WZ_\nu Z^\nu] \tag{4.17}$$

where

$$M^2=\mu_1^2+\mu_2^2+\mu_3^2+\mu_4^2. \tag{4.18}$$

Therefore, in this model, we identify the mass of the W bosons as

$$M_W=Mg/2F. \tag{4.19}$$

From (4.9), (4.16) and (4.18), we can show that

$$M = 4\eta F$$

and hence

$$\eta = M_W/2g \tag{4.20}$$

which is the same as (3.19).

In contrast with the model in § 3, the complete lepton mass term (4.8) in this new model is no longer invariant under the transformation (4.1). On transformation, the term (4.1) becomes

$$\begin{aligned}\tfrac{1}{2}\bar{\psi}_\alpha(-\mu_1\lambda_4-\mu_2\lambda_3)\rho_4 I &- \mathrm{i}\mu_3(\cos 2\alpha\lambda_4\rho_3\gamma_5+\sin 2\alpha\lambda_3\rho_2\gamma_5)\\ &-\mathrm{i}\mu_4(\cos 2\alpha\lambda_3\rho_3\gamma_5+\sin 2\alpha\lambda_4\rho_2\gamma_5)\psi_\alpha+\text{conj}.\end{aligned} \tag{4.21}$$

On decomposition to four-component spinors, the terms involving $\sin 2\alpha$ are zero by virtue of the presence of h_+h_- terms. The remaining terms give

$$-\bar{d}[\mu_1+\mu_2+\cos 2\alpha(\mu_3+\mu_4)]d-\bar{u}[\mu_1-\mu_2+\cos 2\alpha(\mu_3-\mu_4)]u.$$

Taking the value of $\cos 2\alpha$ given by (4.5), we must impose, in addition to (4.11), the conditions

$$\mu_1+\mu_2-\tfrac{1}{3}(\mu_3+\mu_4)=m_\mathrm{d} \tag{4.22a}$$

$$\mu_1-\mu_2-\tfrac{1}{3}(\mu_3-\mu_4)=m_\mathrm{u} \tag{4.22b}$$

where m_d and m_u are the masses of the down and up quark respectively. Solving (4.11) and (4.22) for μ_i, $i=1,2,3,4$, and using the results in (4.18), we find that

$$M^2=[5m_\mathrm{e}^2+9(m_\mathrm{d}^2+m_\mathrm{u}^2)-6m_\mathrm{e}m_\mathrm{d}]/16.$$

If the hypothesis in (3.21) were correct, then the normalisation factors g^2 and F^2 of the different terms (3.23) and (4.17) in the boson Lagrangian would have to be the same. But, from (4.19),

$$g/F=2M_W/M \tag{4.23}$$

which is of the order of 300. As in the model of § 3, we cannot adopt the same normalisation for all the terms.

By changing the mass model of § 3 with the introduction of extra terms, we have been able to use the postulate put forward in an earlier paper that the extended covariant derivative of the first generation quarks can be obtained from that of the leptons by changing the representation of the Clifford algebra. Since the element $\mathrm{i}\Gamma_8\Gamma_9$ and the angle α in the transformation (4.1) are constant, under the change of representation the lepton extended covariant derivative Δ^μ transforms to

$$T_\alpha\Delta^\mu T_\alpha^{-1}.$$

Hence the terms $\{\mathrm{Tr}[g_{\mu\rho}g_{\nu\sigma}M_\mathrm{A}^{\mu\nu}M_\mathrm{A}^{\rho\sigma}]\}/16$, $\{\mathrm{Tr}[g_{\mu\rho}g_{\nu\sigma}M_C^{\mu\nu}M_C^{\rho\sigma}]\}/16$ and $\{\mathrm{Tr}[g_{\mu\rho}g_{\nu\sigma}M_\mathrm{B}^{\mu\nu}M_\mathrm{B}^{\rho\sigma}]\}/16$ appearing in (3.23), (4.15) and (4.17) are the same in both the quark and the lepton representations. So the boson Lagrangian terms are invariant under the change of representation.

5. Summary and discussion

First we shall summarise the previous four sections of this paper.

Section 1 lays down the general principles of spin gauge theories, emphasising the differences between spin and standard gauge theories. Among the important properties of spin gauge theories are their consistency within the Clifford algebra framework, the broader range of allowable gauge transformations, the unification of spacetime and symmetry transformation spaces within the algebraic framework, and the gauge invariance of all possible fermion mass terms.

Section 2 shows how the matrix elements of the GSW theory of electroweak interactions of the electron and its neutrino are the consequence of a particular spin gauge invariance. The leptons form a 16-component spinor, acted on by elements of the Clifford algebra $C_{2,6}$.

Section 3 introduces a new set of concepts concerning mass. These start with the idea that a fermion mass is the result of a coupling of the fermion field to the 'frame field'. The frame field appears naturally in an 'extended covariant derivative' in the same way as the normal boson fields. These ideas are applied to the lepton model of § 2. The commutator of components of this extended covariant derivative gives rise, in 'Dongpei gauge', to boson mass terms; these mass terms have precisely the observed properties of the masses of the photon, W and Z boson fields; the W mass can be fixed by choosing a basic constant η of the frame field, the 'frame inertia'. The extended covariant derivative and frame field therefore give an explanation of boson masses in the GSW theory which does not depend upon the Higgs–Kibble mechanism. Our approach also contrasts with the kinetic view of mass suggested by the 'free electron' operator $(\gamma_\mu p^\mu + Im)$, with the mass m considered to be a 'fifth component' of 4-momentum; the basic difficulty of this approach is that $\{\gamma_\mu\}$ belongs to the odd part of the Dirac algebra, while I belongs to the even part. The frame field concept, with mass based on a vector potential rather than being a 'kinetic scalar', eliminates this algebraic awkwardness.

Section 4 uses a postulate put forward in an earlier paper [4], that the extended covariant derivative for first generation quarks is obtained from that for the leptons by simply changing the Clifford algebra representation, and introducing extra mass terms to fit the masses of the up and down quarks. The correct electromagnetic interactions of the quarks are given by a specific change of the eighth basis vector; we show that this change of representation also gives the correct GSW weak interactions of the first generation quarks. The masses of the W and Z bosons are invariant under the transformation from lepton to quark representation, and the boson mass ratios are unaffected by the inclusion of the extra mass terms in the extended covariant derivative in the model of § 4.

We have therefore been able to formulate a spin gauge theory, introducing mass through the new concepts of the frame field and the extended covariant derivative, which reproduces all of the interactions and the boson masses of the GSW theory. Further study of this theory is needed in order to investigate the alternative to the 'Higgs structure' which follows from the assumption of the physical existence of the frame field, and of the particles which will be associated with it in a quantised theory.

Our proposal to interpret fermion mass terms in terms of the frame field is a philosophical innovation. In our factorisation of mass terms we have written

$$4I_{16} = \sum_{\mu=1}^{4} \Gamma_\mu \Gamma^\mu = \eta^{-1} \lambda_4 \rho_4 \gamma_\mu \phi^\mu. \tag{5.1}$$

The frame field ϕ^μ is thus defined by

$$\begin{aligned} \phi^\mu &= \eta \gamma^\mu \\ &= \eta g^{\mu\nu} \gamma_\nu. \end{aligned} \tag{5.2}$$

In this paper we have kept the metric fixed and constant; the spacetime metric is Minkowski. Therefore,

$$\phi^i = -\eta\gamma_i \qquad i = 1, 2, 3$$

$$\phi^4 = \eta\gamma_4.$$

Thus each component of the frame field is proportional to an element of the spacetime vector basis. Since $\{\gamma_\mu(x)\}$ represent sets of axes at every point of spacetime, the frame field is a physical embodiment of the frame of reference and provides a background for other physical fields. The 'completely empty vacuum' has long been rejected in fundamental physical theories; we now suggest that the frame field is universally present, and is responsible for the 'drag' which massive fields experience, preventing them from travelling with the velocity of light.

In this paper the upper and lower suffix γ-matrices are treated quite differently, and play different physical roles: the $\{\gamma_\mu\}$ are associated with the fermion spinors in the current vector $\bar{\psi}\gamma_\mu\psi$, while the $\{\gamma^\mu\}$ become the frame field. We believe that it is important to generalise these ideas to manifolds with more general metrics; then the different roles of $\{\gamma_\mu\}$ and $\{\gamma^\mu\}$ will become even more distinct. The drag of a massive particle on the frame field could distort the field, and this distortion influence the motion of a neighbouring massive particle; in other words, the frame field might be the carrier of Einstein curvature. It is clear that these intuitive ideas need to be embodied in a carefully constructed mathematical framework before they can be regarded as more than an attractive hypothesis.

In this paper, we have carried out our calculations in Dongpei gauge. This choice of gauge corresponds to a spacetime basis which is constant, and is thus related to a Cartesian frame. Since a spin gauge theory is invariant under a class of gauge transformations, we can imagine that some computations can best be carried out in a gauge in which the γ-matrices are not constant, perhaps (but not necessarily) corresponding to a geometric frame whose axes vary in spacetime.

The details of our calculations have revealed certain restrictions that arise from the use of an extended covariant derivative. We have seen that the preservation of SU(2) symmetry in the W field masses depends critically upon the helicity factors in the GSW interactions, and that, apart from an overall factor, the boson mass matrix is precisely determined by the GSW interactions and the fermion masses. The fact that these restrictions upon the theory produce mass properties which agree exactly with experiment provides theoretical links between the observed properties of the matrix elements and the boson masses. In a more general model, we would expect similar restrictions to arise (a) through the sharing of the group symmetry of matrix elements by the boson masses, and (b) by the dependence of the boson mass matrix on the fermion masses, determining exactly which fields corresponded to observed particles, and fixing their mass ratios. We also suggest that, in a more complete model, the commutator $[\Delta_\mu, \Delta_\nu]$ in Dongpei gauge might be proportional to the complete boson Lagrangian; this is not true in the present models, and would imply yet further restrictions on the masses and coupling constants of a model. A study needs to be made of these restrictions on a general model based on the extended covariant derivative. The hypothesis embodied in (3.21) implies the existence of one or more fermions with mass of the same order of magnitude as $2M_W$ so that a mass relationship of the form (4.23) with $F = g$ can hold; it is doubtful whether the mass of the 'top quark' will be sufficiently large to provide a relationship in a full theory. If this is so, it is possible that a fourth generation of fermions would be sufficiently massive. In formulating a full theory, one also faces the universal problem of deciding which particle fields are fundamental; for example, should the muon be regarded as fundamental, or should it be regarded as a three-particle decaying state? We have assumed that the neutrino mass is zero, but a non-zero mass will still give, through (4.14), a zero photon mass. Also, through (4.16) and (4.18), the relation (4.20) would be maintained. A non-zero mass would of course have some observable physical consequences.

This constant η plays a very significant role in defining the frame field, and its value is very important. At first sight it seems that the value given by our models will be highly model dependent, but this may not be so. In the model of § 3, which contains only one fermion mass, we find from (3.3) and (3.19) the value

$$\eta = M_W/2g. \tag{5.3}$$

This same value is given by (4.20) for the fuller model of § 4, which includes quark masses. It may be that (5.3) is a model-independent formula. If in (5.3) we use the value $\sin\theta_W = 0.5$, then η has the value $M_W/4e$. In a quantised field theory, the coupling constant e can be regarded as dimensionless, so that η has the dimension of mass, and is approximately $3M_W$; η is not the mass of the frame field quanta, which is zero, but we would expect the presence of the frame field to be detectable at energies corresponding to η, which will be accessible with the next generation of machines. We repeat however that the value (5.3) is based upon limited models, not incorporating strong interactions and other generations, and so may not be accurate. In a quantised theory, η would also define a fundamental length; so the value of η is of great importance, but our models need to be extended before the relation (5.3) can be regarded as a definite consequence of this type of theory.

A fundamental problem of theories involving massive bosons is that of renormalisation. The standard GSW theory, based on the Higgs–Kibble mechanism, was eventually shown by 't Hooft [12] to be renormalisable. In this paper we do not attempt to study the renormalisation problem, but we shall explain why we believe that the theory may well be renormalisable. The boson mass terms (3.18) are, in a general gauge, given by (3.9) and (3.16); that is, they are *interaction terms of massless boson fields* of the form '$\phi\phi WW$'. The other boson–boson interaction terms are the $[\phi_\mu, \phi_\nu][\phi^\mu, \phi^\nu]$ type terms in (3.15). So in a general gauge, we are dealing with *massless* fields with interaction terms of an order which might reasonably be expected to be renormalisable. The fact that ϕ_μ is a matrix vector field means, however, that we cannot immediately extend established proofs of renormalisability.

In this paper, we have assumed a constant metric for the eight-dimensional vector space associated with $C_{2,6}$. However, if the associated space had an x-dependent metric, then it would be very natural to extend the gauge transformations (whose generators (2.9) are formed from $\Gamma_5, \Gamma_6, \Gamma_7, \Gamma_8$) to include Lorentz transformations (whose generators are bivectors in the space spanned by $\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4$), and to relate them to gravitation [13–15]. It seems that gravitation may arise naturally when we consider the 'free frame field' Lagrangian, that is, the first term in (3.15), on a curved Riemannian manifold; preliminary calculations (and related work [16–18]) indicate that this term is closely related to the curvature scalar, through the spin connection. It would not be surprising if the frame field turned out to describe gravitation, since the basic Clifford algebra relation

$$\{\gamma_\mu(x), \gamma_\nu(x)\} = 2g_{\mu\nu}(x)I \tag{5.4}$$

holds on a manifold with x-dependent metric, and says that $\gamma_\mu(x)$ is the 'Dirac square root' of the metric. It would be very interesting if the free frame field, arising in our flat-space model, did turn out to describe gravitation on a curved manifold. Normally, terms are introduced into a covariant derivative to ensure covariance of a given quantity under certain transformations; this is not true of the frame field term in (3.6). However, we envisage that on a curved manifold the frame field might be related to Poincaré covariance in spacetime.

References

[1] Chisholm J S R and Farwell R S 1980 *Spin Gauge Theory of Electric and Magnetic Spinors* (*Lecture Notes in Physics* **116**) ed K Osterwalder (Berlin: Springer) p 305; 1981 *Proc. R. Soc.* A **337** 1
[2] Dongpei Z 1980 *Phys. Rev.* D **22** 2027
[3] Wilson W J 1980 *Phys. Lett.* A **75** 172
[4] Chisholm J S R and Farwell R S 1984 *Nuovo Cimento* **82A** 145
[5] Kähler E 1962 *Rendeconti di Math.* **21** 425
[6] McEwan J *Preprint. Spin gauge extensions of electrodynamics*
[7] Chisholm J S R and Farwell R S 1984 *Nuovo Cimento* **82A** 185
[8] Chisholm J S R and Farwell R S 1984 *Nuovo Cimento* **82A** 210
[9] Taylor J C 1976 *Gauge Theories of Weak Interactions* (Cambridge: Cambridge University Press)
Aitchison I J R 1982 *An Informc Introduction to Gauge Field Theories* (Cambridge: Cambridge University Press)
[10] Barut A O and McEwan J 1984 *Phys. Lett.* **135B** 172, **139B** 464; 1986 *Lett. Math. Phys.* **11** 67
[11] Smith F D 1986 *Clifford Algebras and Their Applications in Mathematical Physics* ed J S R Chisholm and A K Common (Dordrecht: Reidel) p 377
[12] 't Hooft G 1971 *Nucl. Phys.* B **35** 167
[13] Weinberg S 1972 *Gravitation and Cosmology* (New York: Wiley)
[14] Utiyama R 1956 *Phys. Rev.* **101** 1597
[15] Kibble T W B 1961 *J. Math. Phys.* **2** 212
[16] Schrödinger E 1932 *Sitzber. Preuss. Akad. Wiss. Phys. Math.* **11** 105
[17] Loos H 1963 *Ann. Phys., NY* **25** 91
[18] Hestenes D and Sobczyk G 1984 *Clifford Algebra to Geometric Calculus* (Dordrecht: Reidel)